On A Certain Game of Solitaire

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Introduction

Consider a card game where you take a shuffled standard deck, and simply draw cards one at a time. At any given point, you are only concerned with the four top cards in your hand. Call the top card “1”, and the fourth card “4”. If the suits of 1 and 4 are the same, then remove cards 2 and 3 so that now 1 and 4 are adjacent in your hand. Then repeat the process. Once you can no longer remove any more cards, draw another card.

This is a straightforward card game, which is deterministic if the deck order is known (as compared to other forms of Solitaire, where choices need to be made at certain points.) The goal of this paper was to determine the distribution of the random variable defined as the number of cards not successfully withdrawn from the hand. We find this random variable approximately follows a normal distribution with mean 26 and standard deviation of 7, so that on average one can expect to remove half the deck, and two-thirds of the time one can expect to remove between 19 and 33 cards.

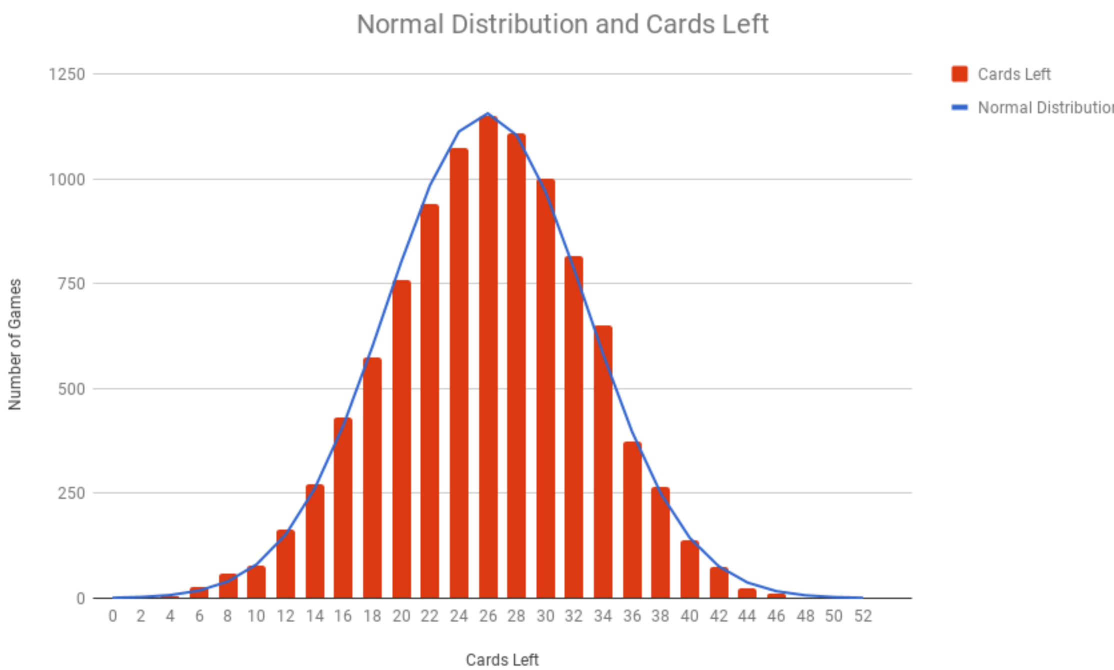
Model

A model was created using Python, in which a deck of cards is shuffled using `random.shuffle()`, then drawn from following the rules above. At the end of each game, the number of cards not successfully withdrawn is recorded. The deck of cards only consists of 13 of each suit, with no other defining factors (numbers and faces are not included), as the only comparison being made is the suit. The code can be found on GitHub.

The sequence of cards left were exported to a text file, and analyzed using spreadsheet software.

Results

The card game was played via simulation 10,000 times. In those trials, the mean number of cards left was 25.9, with a standard deviation of 6.9. Upon looking at a histogram of the distribution (where the “bin size” is 2), it was conjectured a normal distribution may be an appropriate measure for the random variable. Below is a plot of a normal distribution with mean 25.9 and standard deviation 6.9, scaled by 20,000 to match the number of trials and bin size of the histogram.



One can see a very good fit passes the "eye-test." We can compare the percent error at each point. As it turns out, as long as someone stays in the range of 12 to 44 cards left-over, the percent error is less than 10%. Between 18 and 32 cards left, the percent error is less than 5%. This makes sense, as there will likely be a larger cluster toward the mean given a reasonably finite number of trials. Any outliers are unlikely. In addition, there are inherent limitations to this approximation. Indeed, the minimum number of cards that can remain is 2, and naturally the most that can be left is 52 (although our simulations only achieved a maximum of 48.)

Overall, this seems to be a good approximation for the distribution of cards left, which makes this game interesting from a probabilistic standpoint. It also gives the player a good understanding that this is an overall fair game, in a sense. If one earned money on a bet by removing more than half a deck, and lost the same amount by failing to remove more than half, a player would expect to come out even.